Determining the Probabilities of Winning at the Dominoes 42 Game Sevens Using Monte Carlo Simulation

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Abstract

The Dominoes Game of "42" is a 136 year old game played with 28 dominoes. As the 42 game evolved, various game variations emerged. Players who have successfully won a bid can choose to play the basic 42 game or one of several game variations. One of these variations is known as "Sevens". This study proposes an algorithm and then uses Monte Carlo simulation to evaluate the algorithm in predicting the probability of getting and winning a Sevens hand. This is a unique study in that these measures have heretofore not been known. The study results are an aid to any player who wishes to know the odds of getting and winning a Sevens hand. Complete simulation output is presented and some practical rules of thumb based on the simulations are suggested at the end of the paper.

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1. Introduction

a. Origins of the Dominoes Game of 42

According to the Texas State Historical Association, the domino game 42 was developed in 1887 in Parker County, Texas [TSHA]. The game is also known as Texas 42 and is sometimes referenced as the state game of Texas. In 2011, the Texas Legislature passed Bill HCR 84, formally designating 42 as the official State Domino Game of Texas [HCR84]. In 2005, the National 42 Players Association was founded. One goal of the N42PA was to address the variety of rules in 42 tournaments and to sanction player tournaments that played only the standard 42 format [N42PA].

b. The Standard Game of 42

While it is not the objective of this paper to instruct the reader on how to play 42, it is necessary to provide some basic game concepts for the reader who is not familiar with this over one hundred year old game. The game is played using twenty-eight dominoes (0-0 through 6-6) by four players as two teams of two players each. The standard 42 game is a trick-taking game and includes trumps. Teams bid at the beginning of each hand and the team winning the bid is allowed to declare trump. Bridge players would recognize similarities between 42 and contract bridge.

c. Terminology (Game, Hand, Trick)

At the beginning of one hand, the four players each randomly choose seven dominoes from a pile of face-down dominoes that were previously shuffled by one of the team members. After looking at their dominoes, each player gets one chance to enter a bid or pass if they do not wish to bid. Once all players have bid or passed, the winning bidder declares what game variation will be played and what will be trump. The winning bidder leads the first domino and each player plays one domino with the winning domino taking the first trick. The winner of the first trick then begins the second trick by playing a single domino and then each player plays one domino in turn. The winner of this second trick starts the next trick and so on. Once all four players have played all of their dominoes, a total of seven tricks have been played and the score for the hand is calculated based on the number of tricks won and the value of each trick. There are 42 possible points spread across the seven tricks, hence the name of the game. So if the winning team took all seven tricks, their score is 42. If the bidding team's score for the hand is equal to their bid or better, then they are the winner of the hand. The teams continue to play hands until one team reaches the winning score for the

game. In summary, seven **tricks** constitute a **hand** and **hands** are played until one team has a winning score, thus winning the **game**. In this paper, to avoid any confusion, all references to the 42 game variation known as Sevens will be capitalized.

d. Game Variations and the Forced Bid

In addition to "standard" 42, there are a number of 42 game variations that may be declared by the winning bidder. These variations include eighty-four, splash, plunge, nel-o and Sevens [Roberson, p 118]. Each player group generally decides in advance which game variations they will allow and under what circumstances they will be allowed. For example, in N42PA tournaments only standard 42 rules are permitted and no game variations are allowed [N42PA]. Informal player circles typically permit some game variations to be bid, but usually only in the case of a "forced bid".

A forced bid occurs when the first three players have not bid (passed) so if there is to be a bid for this hand, it must come from the fourth bidder. In this case, the fourth bidder must bid and then may choose, for example, "Sevens" if they feel they have a strong "Sevens" hand. Many player groups require that the forced bidder hold at least one domino totaling seven pips (dots) if they select Sevens. While this author cannot find any firm rule for this requirement, it is assumed to be the case since it would not make sense to select Sevens without holding even a single domino with seven pips.

2. The 42 Game Variation "Sevens"

a. Popularity and Rules of Sevens

Because Sevens is not allowed in tournament play and because it differs significantly from standard 42, it is safe to assume that Sevens is not an option played frequently [Roberson, p. 123]. In a hand of Sevens, play starts normally with domino selection. Once the winning bidder chooses Sevens as the play option, play proceeds quite differently from standard 42. To start the hand, the bidder leads their first domino which totals seven pips, for example a 6-1. Then each player, in turn, plays a domino with 7 pips or as close to 7 pips as they can. For example, if a player does not have a 7 pip domino, they must play a domino with pips closest to seven, for example a 4-2 or a 5-3. The first player who played the domino closest to seven pips wins the trick and that player then plays the first domino of the second trick. The domino they play must have seven pips or be as close to seven pips as they can play.

The play continues as long as the winning bidder or their partner continue to take every trick. In the event a player on the opposing team plays the domino closest to seven first, the hand is over and the points for the hand are rewarded to the opposing team.

In Sevens, all players must play their dominoes in order as specified above. Clearly, there is no player strategy involved in playing the Seven hand. In fact, each player can lay their dominos face up at the beginning of the hand for all to see. The outcome of the hand was already determined when the players randomly selected their dominoes.

b. The Away Count

During play of the hand, players use some simple mental math as each domino is examined to determine how far the domino value departs from seven. Players have adopted a shorthand description of each domino which they say as they play the domino. This is the "away" count which is simply the sum of the pips minus seven, expressed without a negative sign. If p1 represents the number of pips on the left side of the domino and p2 represents the number of pips on the right side of the domino, then:

Away Count =
$$|7 - (p1+p2)|$$

For example, as a player plays a 5-5 domino, they would say "3-away". In Sevens, a 6-3 domino is equal to a 3-2 domino in value, as both are "2-away". In this paper, references to an away count are always specific to a single domino.

d. Making Estimates of Winning a Sevens Hand

Clearly, as a player is considering playing Sevens, they make a mental estimate of whether the hand has a good chance of winning. They are looking for mostly sevens and "1-always". Adair [Adair, p 34] has stated "A good Sevens hand would have at least two 7 totals, 2 or more 6 or 8 totals, and 2 or more 5 or 9 totals."

3. Proposed Algorithm for Estimating Game Success

In analyzing the possible success of a seven hand, rules of thumb are helpful but it would be more useful if the player could use some simple metric to accurately estimate the probability of success. It is possible for a player to hold three sevens and four "1-aways" which would be a perfect (unbeatable) hand. At the other extreme, if a

player held two 4-aways, three 5-aways and one each of a 6-away and a 7-away they have the worst possible winning hand. But for the normal situations in between these limits, how do we estimate the relative chances of winning the hand?

In contemplating developing a simple algorithm to use, one would require that it be:

- Simple, easy to explain and
- easy to remember and
- easy to compute and
- accurately predict probability of success

After giving thought to these considerations, it seemed to this author that simply adding up the away counts of the dominoes in the hand would meet the above criteria. The use of this algorithm was selected as the predictive model to be studied and evaluated and is the subject of this paper.

a. The Away Score as a Success Indicator

The algorithm suggested above requires adding up the away counts of all seven dominoes in the hand. In this paper, this sum of individual away counts is referred to as the away score for the hand.

Away Score = Sum of the **Away Counts** of the seven dominoes in the hand.

For example, if the hand contains these dominoes, shown in the order they would be played:

then the algorithm will yield the away score of 10.

$$0 + 1 + 1 + 1 + 2 + 2 + 3 = 10$$

Note that this could be a mental calculation as it requires adding six single digit integers. Later in the paper it will be shown that this away score of 10 predicts a Sevens hand success better than seventy-seven percent, although it contains only one seven (6-1).

Obviously, as the away score increases, the probability of winning the hand decreases. Knowing an away score that yields a success of better than seventy-seven percent would be very useful to the Sevens player.

b. Number of Dominos in Each Away Category

In reviewing the game metrics, knowing all the possible away counts is required. Specifically, how many 1-aways, 2-aways, etc. are there in the 28 dominoes? Simple enumeration provides the following list of away counts for all 28 dominoes.

Away Count	Number of Dominoes
0	3
1	7
2	5
3	5
4	3
5	3
6	1
7	1

TABLE 1: AWAY COUNT FREQUENCY

Studying Table 1, it can be seen that the lowest away score possible in the seven domino hand is 0+0+0+1+1+1+1 totaling 4 and the worst possible score is 4+4+5+5+6+7 which totals 36. So the away score must have a value in the range of 4 to 36. Of course, the odds of getting an away score at either extreme are very small. This study result will permit the player to estimate the probability of winning a Seven hand at each value in the away score range.

4. Deterministic Nature of the Game

As was pointed out previously, once the dominoes are selected by the players, the outcome of the hand is already determined. The players merely have to go through the steps of playing the hand to determine the outcome. Note that the rules do not force the dominoes themselves to be played in a specific order. For example, if a player has to play a 1-away, they may choose to play either their 6-0, their 4-4, or their 5-3. The order of play can be described as deterministic in that the away count values must always be played in a specific and predetermined order. The hand can be described as a stochastic model with a sequence of seven events (tricks) in which the probability of each event (trick) outcome depends only on the state of the dominoes established in the previous trick. Therefore, a Sevens hand can be viewed as a seven state discrete-time Markov chain [Markov].

5. Monte Carlo Simulation

While the outcome of an individual Sevens hand is deterministic, it is difficult if not impossible to calculate the win-probability vs. away score distribution of all possible hands. However, if one were to play a very large number of random hands while keeping track of the outcomes, then one could estimate with good accuracy the probability distribution of win-probability vs. away score. A computational algorithm using repeated random sampling to determine probability distribution estimates is the well-known Monte Carlo method which will be used to construct the models in this study [Monte].

a. Development of Computer Algorithm

To apply the Monte Carlo algorithm to this problem, this study constructed a C++ computer model which paralleled the physical playing of the Sevens hand. The model then played millions of hands using random sampling to initially distribute the dominoes in each hand. The overall organization of the Monte Carlo computer model is summarized below. Appendix C contains the detailed pseudocode.

Establish table of away distributions

Establish the 4 player arrays and other housekeeping variables

Play a single hand

- Randomly distribute dominoes to players
- Sort player holdings by away count
- Loop through to play all seven tricks
- Collect data for this hand

Loop through the indented steps above millions of times

Summarize collected statistical data

b. Decision Points in Constructing the Computer Model

One of the decision points considered was whether to include all possible plays (as in the algorithm above) or whether to limit hands played to situations where the player has to have at least one seven in order to play the hand (required by some player

groups). It was decided to develop models to study both scenarios. The models were designated as:

Type A (plays all shuffles regardless of content), Type B (plays only if the hand contains no sevens) and Type C (only plays shuffles with at least one seven).

The program developed to simulate model Types B or C is identical to the Type A model but reshuffles the dominoes if the starting hand does not have the correct count of seven-pip dominos. For an otherwise identical simulation, the Types B and C model will require significantly more shuffles. Also, the range of possible away scores for Type C is 4 to 32 as now the worst possible hand is 0+4+5+5+6+7. Type B simulations showed that the number of possible wins is always less than five percent so discussions of Type B output are of little interest and are not addressed in this paper.

Another decision is the selection of the random number generator and the random number seed. The random number generator selected was the standard one supplied in the Microsoft Visual Studio 2022 C++ library: srand(seed) and rand(). Experiments with various large random number seeds showed only small differences for large simulations. For example, model Type A was played 5 million times with two different random number seeds: 987,653,131 and 314,159,265. It showed wins of 809,692 and 810,808 respectively, a difference of 0.1%. The seed of 314,159,265 was chosen for subsequent simulation runs.

An additional issue concerns how many games must be played to be sure of convergence onto a reliable solution. A typical approach to this is to experiment by increasing the number of iterations until the solution is seen converging to a single stable value. Experiments showed that 5 million iterations was more than adequate to assure convergence. For example, in a 2 million iteration Type A test, the win rate was 162,008 wins per million whereas in the previous example of a 5 million iteration test, the win rate was 161,938 wins per million, a difference of less than 0.1%. Out of an abundance of caution, all final simulations in this study were run by playing 20 million hands (iterations).

6. Simulation Results

a. Description of Simulation Output

While the simulation is playing the hands, it is collecting considerable data which is of value in analyzing the Sevens game. For each away score, the simulation program was designed to collect:

- the probability of winning with this away score
- the probability of the partner assisting in the win
- the probability that the player has a 6-away or 7-away (1-0 or 0-0 domino)
- the probability that this away score will occur

and for Type C simulations, the probability that any random hand will contain at least one domino containing 7 pips.

Table 2 shows the output for the Type A simulation of 20 million hands:

TABLE 2: TYPE A SIMULATION OUTPUT

						%	
		% Wins				Chances	
Away	%	Partner	%	Winner	Winner	of	Cumul-
Score	Wins	Helping	Losses	Had a 6	Had a 7	this	ative
						Away	%
						Score	
4	100.0	0.0	0.0	0.0	0.00	0.003	0.00
5	100.0	0.0	0.0	0.0	0.00	0.020	0.02
6	99.5	0.2	0.5	0.0	0.00	0.079	0.10
7	97.2	1.2	2.8	0.0	0.00	0.219	0.32
8	93.0	2.8	7.0	0.0	0.00	0.507	0.83
9	86.5	5.1	13.5	0.1	0.00	0.981	1.81
10	77.1	8.0	22.9	0.3	0.00	1.703	3.51
11	64.7	11.3	35.3	0.6	0.00	2.658	6.17
12	50.4	14.3	49.6	1.0	0.00	3.839	10.01
13	35.6	15.7	64.4	1.5	0.04	5.119	15.13
14	24.0	15.2	76.0	1.9	0.21	6.426	21.56
15	16.0	13.3	84.0	2.2	0.60	7.561	29.12
16	11.7	11.1	88.3	2.3	1.19	8.450	37.57
17	9.4	9.4	90.6	2.3	1.77	8.930	46.50
18	8.2	8.2	91.8	2.2	2.16	9.008	55.50
19	7.3	7.3	92.7	2.3	2.40	8.653	64.16
20	6.6	6.6	93.4	2.4	2.62	7.930	72.09
21	5.9	5.9	94.1	2.3	2.79	6.946	79.03
22	5.3	5.3	94.7	2.3	2.91	5.805	84.84
23	4.7	4.7	95.3	2.3	2.86	4.642	89.48
24	4.2	4.2	95.8	2.2	2.76	3.540	93.02
25	3.8	3.8	96.2	2.1	2.72	2.558	95.58
26	3.4	3.4	96.6	2.1	2.68	1.765	97.34
27	3.0	3.0	97.0	2.0	2.56	1.151	98.49
28	2.6	2.6	97.4	1.9	2.37	0.707	99.20
29	2.4	2.4	97.6	2.0	2.29	0.405	99.61
30	2.0	2.0	98.0	1.8	1.99	0.216	99.82
31	1.7	1.7	98.3	1.6	1.66	0.106	99.93
32	1.1	1.1	98.9	1.1	1.05	0.047	99.97
33	0.5	0.5	99.5	0.5	0.51	0.018	99.99
34 25	0.0	0.0	100.0	0.0	0.00	0.006	100.00
35 36	0.0	0.0	100.0	0.0	0.00	0.001	100.00
36	0.0	0.0	100.0	0.0	0.00	0.000	100.00

Descriptions of the columns are as follows. Due to round off, some small values may show as zero.

- 1. The away score
- 2. The % of times this away score will lead to a win
- 3. The % of times the partner contributed to the win. Example, for an away score of 12, there were 50.4% wins of which 14.3% were attributed to the partner helping and the remaining 36.1% were due only to the winning bidder.
- 4. The % losses for this away score = 100 column 2.
- 5. The % of time the winner had a 0-0 domino in their hand
- 6. The % of time the winner had a 0-1 domino in their hand
- 7. The % of times this particular away score will be dealt
- 8. The cumulative % of column 7

Columns 1 through 3 are shown graphically in Figure 1 below.

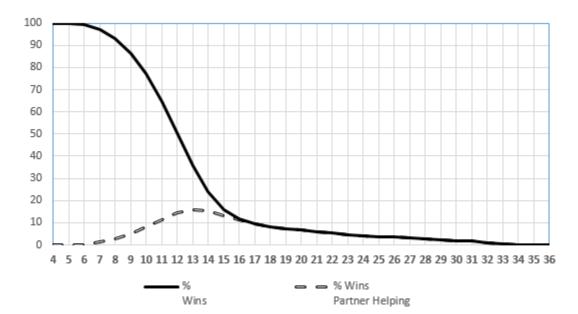


FIGURE 1: TYPE A SIMULATION OUTPUT GRAPHIC

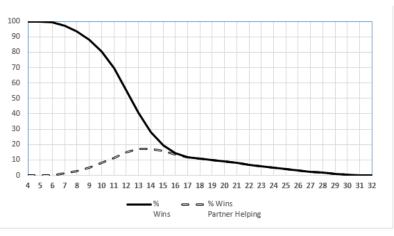
The corresponding graphics for the Type C simulation are shown below. Note that the away score ends at 32 in this case.

TABLE 3: TYPE C SIMULATION OUTPUT

						% Chances	
		% Wins				of	
Away	%	Partner	%	Winner	Winner	this Away	Cumul-
Score	Wins	Helping	Losses	Had a 6	Had a 7	Score	ative %
4	100.0	0.00	0.00	0.00	0.00	0.005	0.005
5	100.0	0.00	0.00	0.00	0.00	0.034	0.040
6	99.4	0.22	0.57	0.00	0.00	0.133	0.173
7	97.2	1.23	2.84	0.00	0.00	0.369	0.542
8	93.5	2.75	6.49	0.00	0.00	0.848	1.390
9	88.1	4.99	11.88	0.08	0.00	1.618	3.008
10	80.2	7.93	19.76	0.30	0.00	2.736	5.744
11	69.4	11.44	30.62	0.64	0.00	4.120	9.864
12	55.7	14.92	44.31	1.16	0.01	5.719	15.583
13	40.6	17.04	59.37	1.82	0.05	7.251	22.834
14	28.2	17.20	71.80	2.42	0.26	8.607	31.442
15	19.3	15.64	80.73	2.88	0.80	9.511	40.952
16	14.5	13.62	85.48	3.15	1.68	9.924	50.877
17	11.9	11.89	88.11	3.26	2.62	9.708	60.584
18	10.8	10.77	89.23	3.15	3.35	9.043	69.627
19	9.8	9.79	90.21	3.40	3.84	7.925	77.552
20	8.9	8.89	91.11	3.53	4.29	6.609	84.161
21	7.9	7.93	92.07	3.49	4.64	5.187	89.348
22	6.9	6.90	93.10	3.31	4.81	3.863	93.211
23	5.9	5.92	94.08	3.31	4.53	2.705	95.916
24	4.9	4.86	95.14	2.95	4.11	1.787	97.703
25	3.9	3.89	96.11	2.75	3.49	1.094	98.797
26	3.2	3.15	96.85	2.54	2.96	0.627	99.424
27	2.1	2.14	97.86	1.81	2.10	0.326	99.751
28	1.7	1.66	98.34	1.55	1.66	0.155	99.906
29	1.0	1.01	98.99	0.98	1.01	0.063	99.969
30	0.3	0.29	99.71	0.29	0.29	0.024	99.993
31	0.0	0.00	100.00	0.00	0.00	0.006	99.999
32	0.0	0.00	100.00	0.00	0.00	0.001	100.000

Columns 1 through 3 are shown graphically in Figure 2 below. Note the partner win contribution increases with higher away scores of 14 or less

FIGURE 2: TYPE C OUTPUT - PARTNER WIN ASSISTANCE VS. AWAY SCORE



The simulations also gather data on how many times a specific away score occurs. Column 7 of Table 3 shows this data which is presented graphically in Figure 3 below. Note that the most likely away score is 16.

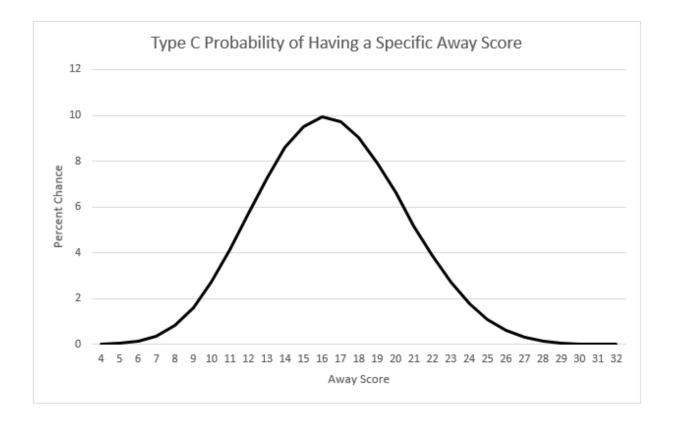


FIGURE 3: TYPE C SIMULATION - PROBABILITY VS. AWAY SCORE

Of additional interest is that the Type C simulation required 33,650,467 shuffles to obtain the acceptable 20,000,000 "good" shuffles. So 13,650,467 additional shuffles were required to satisfy the requirement of "at least one seven domino" hand.

b. Observations and Application of Results

One of the first questions to consider is when to use to use the Type A results and when to use the Type C results. It would not be wise to decide to play Sevens before looking at your hand (Type A) because it might contain no sevens. As previously pointed out, a hand with no sevens has less than a five percent chance of winning. Therefore, once the player looks at their hand they will consider playing Sevens only if they have at least one seven (Type C). In this case, the player always faces competition containing at least one less 7-pip domino, hence one would expect their probability of

winning for any given away score to be higher, and in fact it is for all but the highest away scores. Table 4 shows the Type A vs. Type C comparison.

TABLE 4: TYPE A VS. TYPE C WIN PROBABILITY

Away	Type	Type	Type C
Score	A %	C %	% / YPC C
30010	Wins	Wins	Increase
4	100.0	100.0	0.0
5	100.0	100.0	0.0
6	99.5	99.5	0.0
7	99.5 97.2	99.5 97.2	0.0
8	93.0	93.5	0.0
9	93.0 86.5	93.5 88.1	1.9
9 10	77.1	80.2	4.1
10	64.7	69.4	7.2
12	50.4	55.7	10.5
13	35.6	40.6	14.2
13 14	24.0	28.2	14.2
15	16.0	19.3	20.6
15 16	16.0	19.3	20.6
16 17	9.4	14.5 11.9	23.9 27.1
17 18			
	8.2	10.8	30.9
19	7.3	9.8	33.9
20	6.6	8.9	34.3
21	5.9	7.9	34.9
22	5.3	6.9	30.6
23	4.7	5.9	25.5
24	4.2	4.9	16.3
25	3.8	3.9	2.8
26	3.4	3.2	-8.1
27	3.0	2.1	-29.3
28	2.6	1.7	-37.0
29	2.4	1.0	-58.4
30	2.0	0.3	-85.9
31	1.7	0.0	-100.0
32	1.1	0.0	-100.0
33	0.5		
34	0.0		
35	0.0		
36	0.0		

Note that at low away scores the win probabilities are very close and this is expected since the player holds all the sevens in both cases. As the away score increases, the probability of the competition having more sevens and 1-aways is increasing and therefore the win probability decreases.

In the output, the away score of 5 always shows 100 percent wins. Is this right? With an away score of four the answer is obvious but will an away score of five always guarantee a win or is the program working incorrectly? There are 2 ways to achieve an away score of 5: 0001112 or 0011111. Since there are only seven 1-away dominoes, in the worst case an individual opposing player can have at most four 1-aways. If a single opposing player has all four 1-aways plus a seven domino, their last

play of a 1-away is in the fifth round but the bidding player will always be able to play a 1-away in the sixth round. So yes, an away score of 5 guarantees a win one hundred percent of the time. Sometimes simulations can predict outcomes that were not obvious or apparent!

The Type C simulation required 33,650,467 shuffles to obtain the acceptable 20,000,000 "good" shuffles (Appendix B). So a playable Sevens hand (should have at least one seven pip domino to play) occurs 20,000,000/33,650,467 = 59.44 percent of the time. The breakdown of choosing a hand with some sevens is:

•	A hand with no sevens:	40.56%
•	A hand with one seven:	44.90%
•	A hand with two sevens:	13.47%
•	A hand with three sevens:	1.07%
•	A hand with at least one seven:	59.44%

Table 4 shows that if a player wants to guarantee a better than 50-50 chance of winning, they should play if their away score is 12 or better. As stated previously, one author [Adair, p 34] has suggested "A good Sevens hand would have at least two 7 totals, 2 or more 6 or 8 totals, and 2 or more 5 or 9 totals". So this kind of hand, for example, could have an away score of 0+0+1+1+1+2+2=7 or it could have an away score of 8. From Table 4, these away scores predict a win of 97% or 93% of the time, which is of course an excellent winning hand.

However, consider a hand with these dominoes:

Upon casual inspection, this hand might seem like a sure losing Sevens hand. However, it's away score of 12 says it should win at least half the time, often with a partner's help. In fact, Table 3 shows overall wins 55.7% of the time of which 14.9% were due to the partner's help and the other 40.8% solely due to the player's hand. In other words, if you play a hand with a 12 away score, your partner will assist you in the win 27 percent of the time. It seems likely that players underestimate the success possibilities of hands with higher away scores or the likelihood that their partner can contribute to the win.

Obviously, players wishing to improve their scores will not memorize Table 4. However, this simple rule of thumb is suggested:

- If the away score is 10 or less, there is a more than an 80% win chance so you might consider a stronger bid (2 marks).
- If the away score is 11 or 12, there is a more than a 56% win chance.
- If the away score is more than 16, there is less than a 10% win chance.

An even simpler mnemonic would be: "if you can add up your away score on your fingers, go for it"!

As can be seen in Figure 2, the chances of winning start to drop rapidly once the away score exceeds 10. From Table 2, the probabilities of drawing these winning hands are as follows:

- There is a 3.51% chance of drawing a hand with an away score <=10
- There is a 10.01% chance of drawing a hand with an away score <=12
- There is a 62.43% chance of drawing a hand with an away score >16

7. Conclusions

This study has proposed and evaluated an algorithm which allows players of the 42 game variation Sevens to determine the probability of winning their Sevens hand. This algorithm was tested using Monte Carlo simulation and the results have produced accurate and previously unknown win probabilities for the Sevens game. The simulation results have practical applications as the algorithm is easy to remember and apply by making a simple mental calculation as previously described. With this algorithm, players of 42 can improve their scores by allowing them to select the Sevens option only when it has the best chance to win.

The suggested algorithm was evaluated using a Monte Carlo computer simulation using extremely large sample sizes. Detailed program output statistics are presented here which can be used in many different scenarios to analyze the game and possible outcomes. It is suggested that future authors writing on the game of 42 incorporate some of this information into their texts.

Appendix A

Simulation Technical Details

Development Environment: Microsoft Visual Studio 2022 C++, Version 4.8.09037.

Programming Language: C++ and primarily integer variables for speed.

Random Number Generator: Standard C++ library: srand(seed) and rand().

Random Number Seed: 314,159,265 for all published simulations.

Type A Program Source Code: 263 lines including spacing and comments.

Type C Program Source Code: 278 lines including spacing and comments.

Program Run Time, Type A 20M: 3 minutes 15 seconds (9.8 seconds/million).

Program Run Time, Type C 20M: 5 minutes 25 seconds (16.2 seconds/million).

CPU Processor & Speed: Intel(R) Core(TM) i7-2600 CPU @ 3.40GHz.

Installed RAM Memory 16 GB.

Appendix B

Raw Program Output, Type C Simulation: 20 Million Iterations.

```
STARTING SIMULATION
Number of plays? 20000000
RESULT SUMMARY
NumPlays = 20000000 seed = 314159265
Total Shuffles = 33650467 bad shuffles = 13650467
Percent bad shuffles = 40.5655
Wins = 4825048 losses = 15174952
Points Wins Partner Losses Had6 Had7 Games
   0
           0
               0
                   0 0
   0
       0
           0
               0
                   0 0
   0
       0
           0
                   0 0
   0
       0
           0
               0
                   0 0
3
   1021
          0
              0
   6888
              0
                      0 7909
5
          0
                  0
   26541
           59
                153
                      0
   71684
           911
                 2099
                        0
                            0 108386
                   11017
   158613
            4657
                           0
                               0 278016
            16154
                    38435
                                  0 601627
   285176
                            246
10
    439058
             43372
                     108109
                              1664
                                     0 1148794
             94245
                     252327
11
    571642
                              5251
                                     0 1972763
             170621
                     506848
                               13309
12
    636980
                                       59 3116591
    589193
             247166
                     861040
                               26437
                                       724 4566824
13
             296031
                                        4525 6288315
14
    485525
                     1235966
                                41704
             297521
                     1535558
                                54798
                                        15195 8190480
15
    366607
16
    288269
             270387
                     1696606
                                62526
                                        33328 10175355
    230800
             230800
                                        50810 12116877
17
                     1710722
                                63291
18
    194855
             194855
                     1613698
                                56879
                                        60556 13925430
19
    155143
             155143
                      1429867
                                53823
                                        60931 15510440
    117575
                      1204258
20
            117575
                                46651
                                        56717 16832273
21
    82252
            82252
                    955115
                             36185
                                     48104 17869640
22
    53294
            53294
                    719289
                             25534
                                     37175 18642223
23
    32020
            32020
                    508965
                             17897
                                     24521 19183208
24
    17389
            17389
                    340068
                             10555
                                     14699 19540665
25
    8516
           8516
                  210296
                           6018
                                  7644 19759477
26
    3954
           3954
                  121458
                          3184
                                  3718 19884889
27
    1397
           1397
                  63899
                          1180
                                 1371 19950185
                             516 19981256
28
    516
          516
                30555
                        482
29
    126
          126
                12395
                        123
                              126 19993777
                         14 19998613
30
              4822
         14
                     14
            1125
                      0 19999738
31
        0
                   0
32
                     0 20000000
                  0
                    0 20000000
33
                0
                   0 20000000
34
        0
            0
                0
35
            0
                0
                    0 20000000
            0 0 0 20000000
Total Wins = 4825048 Total Losses = 15174952
Points Wins Partner Losses Had6 Had7
                     0 0.005105 0.005105
             0
                0
  100
             0
                 0
                    0 0.03444 0.039545
```

```
99.4268
              0.221023
                        0.573162
                                    0 0 0.13347 0.173015
7
    97.1552
                                     0 0.368915 0.54193
              1.2347
                       2.84483
                                 0
8
    93.5053
              2.74539
                        6.49472
                                  0
                                      0 0.84815 1.39008
                      11.8769
    88.1231
              4.9918
                                 0.0760172
                                             0 1.61805 3.00813
9
    80.2421
10
              7.92665
                         19.758
                                 0.304112
                                             0 2.73584 5.74397
    69.3766
              11.4379
                         30.6234
                                  0.637281
                                             0 4.11984 9.86382
11
    55.6884
              14.9167
12
                         44.3116
                                   1.16355
                                             0.00515812 5.71914 15.583
13
    40.6275
              17.0432
                         59.3725
                                   1.82295
                                             0.049923 \quad 7.25116 \quad 22.8341
    28.2037
              17.1962
                         71.7962
                                   2.42255
                                             0.262854
                                                       8.60746 31.4416
14
15
    19.2731
              15.6412
                         80.7269
                                   2.88082
                                             0.798827
                                                       9.51083 40.9524
16
    14.5233
              13.6224
                         85.4767
                                   3.15012
                                             1.6791 9.92438 50.8768
    11.8876
              11.8876
                         88.1124
                                             2.61702 9.70761 60.5844
17
                                   3.25987
18
    10.7741
              10.7741
                         89.2259
                                   3.145
                                          3.34831 9.04276 69.6272
19
    9.78814
              9.78814
                         90.2119
                                   3.39575
                                             3.8442 7.92505 77.5522
    8.89485
              8.89485
                         91.1052
                                   3.52927
                                             4.29078 6.60916 84.1614
20
21
    7.92892
              7.92892
                         92.0711
                                   3.48816
                                             4.63712 5.18684 89.3482
    6.89816
                         93.1018
                                  3.30502
                                             4.81178
22
              6.89816
                                                      3.86292 93.2111
23
    5.91883
              5.91883
                         94.0812
                                   3.30822
                                             4.53266
                                                      2.70493 95.916
24
    4.86464
              4.86464
                         95.1354
                                   2.9528
                                            4.1121 \quad 1.78728 \ 97.7033
25
    3.89193
              3.89193
                         96.1081
                                   2.75031
                                             3.49341 1.09406 98.7974
    3.15281
                                   2.53883
              3.15281
                         96.8472
                                             2.96463
                                                      0.62706 99.4244
26
27
    2.13949
              2.13949
                         97.8605
                                   1.80716
                                             2.09967
                                                      0.32648 99.7509
28
    1.66071
              1.66071
                         98.3393
                                   1.55129
                                             1.66071
                                                      0.155355 99.9063
29
    1.00631
              1.00631
                         98.9937
                                   0.98235
                                             1.00631
                                                      0.062605 99.9689
    0.289495
30
               0.289495
                           99.7105
                                     0.289495
                                                0.289495
                                                          0.02418 99.9931
31
        0
            100
                   0
                      0
                           0.005625 99.9987
                      0 0.00131 100
        0
             100
                   0
Total Wins = 4825048 Total Losses = 15174952
```

% of total games won = 24.1252Run Time: 5 minutes, 25 seconds

END OF PROGRAM

Appendix C: Simulation Type A Pseudocode

```
int main()
       initialize domino off-value array
       initialize statistical arrays
       input seed
       input num plays
       for num plays do
               shuffle()
               play_game()
       end
       show_results()
end
void shuffle()
       for 0 to 27 do
               load shuf dom randomized array using rand()
       end
       for player 0 to 3 do
               for domino 0 to 6 do
                      load player hand from shuf dom
               end
       end
       for player 0 to 3 do
               ascending sort player hand
       end
end
int play_game(boolean)
       initialize hand statistics
       assign player as bidder
       for hand 0 to 6 do
               if bidder is playing do
                       find lowest domino
                      if lowest domino is opponent stats(0) & return(fail)
                       if lowest domino is partner assign player as partner
               end
               else if partner is playing do
                       find lowest domino
                       if lowest domino is opponent stats(0) & return(fail)
                      if lowest domino is bidder assign player as bidder
       end
       end
       stats(1)
       return(success)
end
void stats(int)
       if loss do update loss sats
       else update win stats
void show_results()
       display input summary
       for away_count 4 to 32 do
               display stats
       end
       display simulation summary
end
```

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